

### Tutorial 4 - Sheet 3

1. State whether the following functions are one-to-one or many-to-one.

(a)  $y = x^6 + 1$

(b)  $y = ax + b$ ,  $a, b$  constants

(c)  $y = \frac{3}{t-1}$

(d)  $y = x^2 - 4x + 3$   $1 \leq x \leq 4$

(e)  $y = x^2 - 4x + 3$   $0 \leq x \leq 2$

2. The function  $f(x)$  is defined by  $f(x) = x^k$  where  $k$  is a positive integer. State the values of  $k$  for which  $f$  is

(a) a one-to-one function (b) a many-to-one function

3. Given  $f(t) = 2t - 1$ ,  $g(t) = \frac{1}{t+1}$  and  $h(t) = t^2 + 1$  find

(a)  $f(g(t))$  (b)  $h(h(t))$  (c)  $g(f(h(t)))$

4. Find the inverse of the following functions if possible

(a)  $f(x) = 5 - 4x$

(b)  $f(x) = \frac{1}{x+2}$

(c)  $f(x) = \frac{x}{x+2}$

(d)  $f(x) = 6 - x + x^2$

(e)  $f(x) = x^2 + x^3$

5. The function  $f(x)$  is defined by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1; \\ 1 & 1 < x \leq 2; \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch  $f(x+1)$

(b) Sketch  $f(x-1)$

(c) Sketch  $f(x)+2$

6. The function  $M(t)$  is defined by

$$M(t) = \begin{cases} 2, & -1 \leq t < 1; \\ 2-t, & -1 \leq t \leq 2. \end{cases}$$

State (a)  $\lim_{t \rightarrow 0^-} M(t)$  (b)  $\lim_{t \rightarrow 0^+} M(t)$  (c)  $\lim_{t \rightarrow 1^-} M(t)$ .