

## MATHEMATICS FOR AERO/MECHANICAL ENGINEERS

### TUTORIAL SHEET 4

#### ORDINARY DIFFERENTIAL EQUATIONS IV

A

Find the values of the constants for which the given functions are particular integrals of the following differential equations:

- |                                                     |                                            |
|-----------------------------------------------------|--------------------------------------------|
| 1. $ax + b$ ; $y'' + y = 2x$                        | 2. $ae^{bx}$ ; $y'' + 2y' - 8y = 2e^{-2x}$ |
| 3. $a$ ; $y'' - 5y' - 6y = 12$                      | 4. $ax^2 + bx + c$ ; $y'' - 4y = x^2 - 3$  |
| 5. $a \sin px + b \cos px$ ; $y'' + 9y = 2 \cos 2x$ |                                            |

B

Obtain particular integrals of the following differential equations and write down the general solutions:

- |                                      |                                     |
|--------------------------------------|-------------------------------------|
| 1. $y'' - 9y = 2e^{2x}$              | 2. $4y'' - 3y' - y = 2$             |
| 3. $y'' + y = 3 \cos 2x$             | 4. $4y'' - y = x^2 - 3x$            |
| 5. $y'' - 6y' - 16y = e^{-x}$        | 6. $y'' + 2y' + y = \sin 3x$        |
| 7. $2y'' + y' - 3y = \frac{1}{2}x^3$ | 8. $y'' - y' + y = \cos x - \sin x$ |
| 9. $y'' - 4y = e^{-2x}$              | 10. $y'' + 2y' + 5y = 15$           |

C

1. Find the solution of the differential equation which models the spring-mass system with  $m = 1$ ,  $k = 9$  under the influence of an external force  $F(t) = F_0 \cos \omega t$ , with  $F = 80$  and  $\omega = 5$  and given the initial conditions  $x(0) = x'(0) = 0$
2. Find the transient and steady periodic solutions of the differential equation:

$$x'' + 2x' + 2x = 20 \cos 2t \quad \text{with} \quad x(0) = x'(0) = 0$$