MATHEMATICS FOR AERO/MECHANICAL ENGINEERS

TUTORIAL SHEET 6

LAPLACE TRANSFORMS

Solve the following differential equations using the Laplace transform method.

1	$\dot{x} + 3x = e^{-2t}$	$x(0) = 2$	**2.**	$3\dot{x} - 6x = \sin 2t$	$x(0) = 1$
3.	$\ddot{x} - 4\dot{x} + 3x = 0$	$x(0) = 2, \dot{x}(0) = 1$	**4.**	$\ddot{x} + 36x = 0$	$x(0) = -1, \dot{x}(0) = 2$
5.	$\ddot{x} + 4x = t$	$x(0) = 1, \dot{x}(0) = 0$	**6.**	$\ddot{x} - 2\dot{x} + x = e^{t}$	$x(0) = -2, \dot{x}(0) = -3$
7.	$\ddot{y} - 3\dot{y} + 2y = \sin t$	$y(0) = 0, \dot{y}(0) = 1$	**8.**	$\ddot{x} - 16x = t.e^{t}$	$x(0) = \dot{x}(0) = 0$
9.	$\ddot{x} + x = \cos t$	$x(0) = 1, \dot{x}(0) = 2$	**10.**	$\ddot{x} - 6\dot{x} + 8x = 2$	$x(0) = \dot{x}(0) = 0$
11*.	$\ddot{x} + 4\dot{x} + 8x = e^{-t}$	$x(0) = \dot{x}(0) = 0$	**12*.**	$\ddot{x} - 2\dot{x} + x = t.e^{t}$	$x(0) = 1, \dot{x}(0) = 0$

(* not for the faint-hearted)

Section B**

Problems 1 and 2 illustrate two types of resonance in a mass-spring-dashpot system with given external force ***F*(*t*)** and with initial conditions ***x*(*0*) = *x*(*0*) = *0*.**

1. Suppose that ***m*** = 1, ***k*** = 9, ***c*** = 0, and ***F*(*t*)** = *6cos3t*. Use the inverse transform $\left\{ \frac{s}{\left(s^2 + k^2\right)^2} \right\} = \frac{1}{2k} t.\sin kt$ to derive the solution $x(t) = t \sin 3t$.

2. Suppose that ***m*** = 1, ***k*** = *9.04*, ***c*** = *0.4* and ***F*(*t*)** = $6.e^{-t/5}$ *cos3t*. Derive the solution

$$x(t) = t.e^{-t/5} \sin 3t$$