SHEET-1 REVISION OF DIFFERENTIATION

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in each of the following:

(a) $y = 3\sin(2x-1),$ (b) $y = 4\cos(2x+1),$ (c) $y = 2x^5 + e^{5x} - 3,$

(d) $y = 3e^{(5x-1)} + \sin(3x+2) + \cos(4x-1) + 2.$

2. The product rule for differentiation says the if $y = uv,\ then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}.$ Use the product rule to find the first and second derivatives in each of the following:

(a) $y = e^x(3x^5 + 2x^2 - 1),$ (b) $y = 3e^{3x}\sin(3x-1),$ (c) $x = 3\sin(3t-1)e^{(2t-1)}.$

3. A particle moves along the x axis so that its displacement x from the origin at tome t is given by

$$x = 5e^{-3t}\sin 2t.$$

Find the velocity and acceleration at time t and show that

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 0.$$

4. Verify that the following are correct.

(a) $If\ x = A\cos 2t + B\sin 2t,\ then\ \frac{d^2x}{dt^2} + 4x = 0,$

(b) $If\ x = A\cos nt + B\sin nt,\ then\ \frac{d^2x}{dt^2} + n^2x = 0,$

(c) $If\ x = Ae^{3t} + Be^{-3t},\ then\ \frac{d^2x}{dt^2} - 9x = 0,$

(d) $If\ x = e^{-t}(A\cos t + B\sin t),\ then\ \frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0,$

(e) $If\ y = Ae^x + Be^{-x} + C\cos x + D\sin x,\ then\ \frac{d^4y}{dx^4} - y = 0.$

5. Find A and B in each of the following using the information given.

(a) $y = Ae^{-3x} + Be^{-x},\ given\ y = 1\ and\ \frac{dy}{dx} = 2\ at\ x = 0,$

(b) $y = e^{-2x}(A\cos x + B\sin x),\ y = 0,\ \frac{dy}{dx} = 2\ at\ x = 0.$