

SHEET 10 SYSTEMS OF FIRST ORDER DIFFERENTIAL EQUATIONS AND FOURIER SERIES

1. Solve the following systems of differential equations using Laplace transforms

$$(a) \quad \left. \begin{aligned} 2 \frac{dy}{dt} - 6y + 3x &= 0, \\ 3 \frac{dx}{dt} - 3x - 2y &= 0. \end{aligned} \right\} x = 1, y = 3 \text{ at } t = 0.$$

$$(b) \quad \left. \begin{aligned} \frac{dy}{dt} + 3x &= e^{-2t}, \\ \frac{dx}{dt} - 3y &= e^{2t}. \end{aligned} \right\} x = 0, y = 0 \text{ at } t = 0.$$

2. Find the Fourier series of the following periodic functions

$$(a) \quad \left. \begin{aligned} f(t) &= t \quad (0 < t < 2\pi) \\ f(t) &= f(t + 2\pi) \end{aligned} \right\}$$

$$(b) \quad f(t) = \begin{cases} -1 & (-\pi < t < 0) \\ 1 & (0 < t < \pi) \end{cases}$$

$$f(t) = f(t + 2\pi)$$

Answers

$$1. \quad (a) \quad x = 2e^{2t} + te^{2t}, y = \frac{1}{2} \{6e^{2t} + 3te^{2t}\},$$

$$(b) \quad x = \frac{1}{13} (2e^{2t} + 3e^{-2t} - 5(\cos 3t - \sin 3t)),$$

$$y = \frac{1}{13} (5(\cos 3t + \sin 3t) - 3e^{2t} - 2e^{-2t})$$

$$2 \quad (a) \quad f(t) = \pi - 2 \left(\sin t + \frac{\sin 2t}{2} + \frac{\sin 3t}{3} + \dots \right) = \pi - \sum_{n=1}^{\infty} \frac{2}{n} \sin nt.$$

$$(b) \quad f(t) = \frac{4}{\pi} \left(\sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots \right) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)t}{2n-1}.$$