SHEET 2 FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS

1. Write down the complementary function of each of the following linear first order differential equations. Then use the information given to find the arbitrary constant.

$(a)\ \frac{dy}{dx}+2y=0,\quad y(o)=4,\ (b)\ \frac{dy}{dx}+4y=0,\quad y(0)=1,\ (c)\ \frac{dx}{dt}+x=0,\quad x(1)=2,$

$(d)\ 2\frac{dx}{dt}+6x=0,\quad x(0)=2,\ (e)\ 2\frac{dy}{dx}+7y=0,\quad y(0)=2.$

2. Find the solution of the following linear first order differential equations.

To find the particular integral try something of the form $y=\alpha$

$(a)\ \frac{dy}{dx}+2y=12,\ y(0)=2,\qquad (b)\ 4\frac{dx}{dt}=3x+8,\ x(0)=1,\quad (c)\ \frac{dx}{dt}+3x=1,\ x(0)=4,$

$(d)\ \frac{dy}{dx}+4y=9,\ y(0)=2,\qquad (e)\ 4\frac{dy}{dx}+12y=4,\ y(1)=2.$

3. Find the general solution of each of the following linear first order differential equations.

To find the particular integral try something of the form $y=\alpha x+\beta.$

$(a)\ \frac{dx}{dt}+2x=2t+1,\qquad (b)\ \frac{dy}{dx}+4y=-x+1,\qquad (c)\ \frac{ds}{dt}+4s=2t-1,$

$(d)\ \frac{dy}{dx}+9y=3x-1,\qquad (e)\ 2\frac{dy}{dx}+4y=-x+1,\qquad (f)\ 4\frac{dx}{dt}=2x+3t-1.$

4. Find the general solution of the following linear first order differential equations.

To find particular integral try something of the form $y=\alpha e^{mx}$.

$(a)\ \frac{dy}{dx}+2y=4e^{4x},\qquad (b)\ \frac{dx}{dt}+4x=2e^{t},\quad (c)\ 2\frac{dy}{dx}=4y+7e^{-6x}.$

5. Solve the following linear first order differential equations.

$(a)\ \frac{dy}{dx}+2y=3x+1,\quad y(0)=2,\qquad (b)\ \frac{dx}{dt}=-2x+e^{3t},\quad x(0)=1,$

$(c)\ \frac{dy}{dx}+4y=-x+1,\quad y(0)=1,\qquad (d)\ 2\frac{dx}{dt}=-x+2e^{t},\ x(0)=2.$