SHEET 4 THE EULER METHOD

The Euler method is used to approximate the solution of first order differential equations of the form

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0, \quad i.e. \quad y = y_0 \quad at \quad x = x_0.$$

This is done using the Euler algorithm

$$y_{n+1} = y_n + hf(x_n, y_n), \quad x_n = x_0 + nh,$$

where h is the step size.

1. Use the Euler method to obtain approximations to the following differential equation between x=0.0 and x=1.0, using

   (a) A step size h = 0.25,
   (b) A step size h = 0.1.

$$\frac{dy}{dx} = y + 3x, \quad y(0) = 1, \quad i.e.\ y_0 = 1.0, \quad x_0 = 0.0.$$

2. Use the Euler method to obtain approximations to the following differential equation between x=1.0 and x=2.0, using

   (a) A step size h = 0.2,
   (b) A step size h = 0.1.

$$\frac{dy}{dx} = 2y - 3x, \quad y(1) = 1, \quad i.e.\ y_0 = 1.0, \quad x_0 = 1.0.$$

3. Use the Euler method to obtain approximations to the following differential equation between x=0.0 and x=1.0, using

   (a) A step size h = 0.2,
   (b) A step size h = 0.1.

$$\frac{dy}{dx} = 2 - 3y, \quad y(0) = 1, \quad i.e.\ y_0 = 1.0, \quad x_0 = 0.0.$$

Show that the exact solution is given by $y = \frac{e^{-3x}}{3} + \frac{2}{3}$ and compare the exact solution and the approximate solutions. Which of (a) and (b) is more accurate?