SHEET 5 FIRST ORDER DIFFERENTIAL EQUATIONS, APPLICATIONS AND THE EULER METHOD

1. A simple electrical circuit consists of a resistor in series with an inductance. When the switch is closed, a current *i* will start to flow in the circuit. The rate of increase of current with respect to time is given by

$$\frac{di}{dt} = 1 - 2i.$$

Obtain an expression for the current i in terms of *t*, given that $i = 0A \ when \ t = o$. Also sketch a graph of *i against t* and state what the steady state (long time) value of i is. Finally, solve the equation approximately between t = 0 and 1, using the Euler method with a step size h = 0.2, and compare the exact solution and the numerical one.

2. The differential equation governing current flow *(i)* in a series *RL* circuit is given by the following linear first order differential equation

$$iR + L\frac{di}{dt} = t, \quad t \geq 0, \ i(0) = 0.$$

(a) Show that

$$i = \frac{t}{R} + \frac{L}{R^2}\left\{ e^{-\frac{Rt}{L}} - 1 \right\}.$$

(b) Given that *R=10.0, L=5.0* calculate approximations for *i*, at values of *t* between 0 and 5 with a step size *h=0.25* using the Euler method.

(c) Compare the exact solution with the numerical one.

3. Use the Euler method with a step size $h = 0.1$ to obtain approximations to the following first order differential equation between $x = 0 \ and \ x = 1.0$.

$$\frac{dy}{dx} = \frac{x}{1-y}, \ x = 0 \ at \ y = 0, \ i.e. \ x_0 = 0, y_0 = 0.$$

4. Use the Euler method to estimate $y(0.5)$ given that

$$\frac{dy}{dx} = x + y, \ y(0) = 0.$$

Use both a step size $h = 0.25 \ and \ h = 0.1$. Also find the true solution and compare this with both the numerical ones.