

SHEET 7 FURTHER SECOND ORDER DIFFERENTIAL EQUATIONS

1. Solve the following second order differential equations

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x = 0, y = 1$ and $\frac{dy}{dx} = -2$,

(b) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-3t}$, given that at $t = 0, x = \frac{1}{2}$ and $\frac{dx}{dt} = -2$,

(c) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = \sin t$, given that at $t = 0, x = 0$ and $\frac{dx}{dt} = 1$,

(d) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3\sin x$, given that at $x = 0, y = -0.9$ and $\frac{dy}{dx} = -0.7$,

(e) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 85\sin 3t$, given that at $t = 0, x = 0$ and $\frac{dx}{dt} = -20$.

Answers

(a) $y = e^{-2x}(2 - \cos x)$,

(b) $x = \frac{1}{2}(1-t)e^{-3t}$,

(c) $x = -\frac{1}{2}e^t + \frac{1}{5}e^{2t} + \frac{1}{10}(\sin t + 3\cos t)$,

(d) $y = e^{-2x} - e^{-x} + \frac{3}{10}(\sin x - 3\cos x)$,

(e) $x = e^{-t}(6\cos t + 7\sin t) - 6\cos 3t - 7\sin 3t$.