SHEET 9: Solutions of O.D.E.s with constant coeffs

By LAPLACE TRANSFORM METHOD

Solve the following subject to the given conditions.

First order

(1) $3\frac{dx}{dt} - 4x = e^{2t}$; $x = 0$ when $t = 0$.

(2) $3\frac{dx}{dt} - 4x = e^{2t}$; $x = 5$ when $t = 0$.

(3) $3\frac{dx}{dt} - 4x = \cos 2t$; $x = 0$ when $t = 0$.

(4) $3\frac{dx}{dt} - 4x = \sin 2t$; $x = 2$ when $t = 0$.

(5) $\frac{dx}{dt} - 2x = 3t$; $x = 0$ when $t = 0$.

(6) $\frac{d\theta}{dt} = -k(\theta - \alpha)$; where k, α are known constants and $\theta = 0$ when $t = 0$.

Second Order

(1) $\frac{d^2x}{dt^2} + x = \cos 3t$; $x_0 = 0, \left(\frac{dx}{dt}\right)_0 = 0$.

(2) $\frac{d^2x}{dt^2} + x = \cos 3t$; $x_0 = 1, \left(\frac{dx}{dt}\right)_0 = 2$.

(3) $4\frac{d^2x}{dt^2} + 9 = \sin 2t$; $x_0 = 1, \left(\frac{dx}{dt}\right)_0 = 2$.

(4) $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^t$; $x_0 = 1, \left(\frac{dx}{dt}\right)_0 = 1$.

(5) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{2t}$; $x_0 = 0, \left(\frac{dx}{dt}\right)_0 = 0$.

(6) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0$; $x_0 = 1, \left(\frac{dx}{dt}\right)_0 = 0$.

(7) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = \sin t$; $x_0 = 0, \left(\frac{dx}{dt}\right)_0 = 0$

Answers

First order:

(1) $x = \frac{1}{2}\left(e^{2t} - e^{\frac{4}{3}t}\right)$ (2) $x = \frac{1}{2}\left(9e^{\frac{4}{3}t} + e^{2t}\right)$

(3) $x = \frac{-1}{13}\cos 2t + \frac{3}{26}\sin 2t + \frac{1}{13}e^{\frac{4}{3}t}$ (4) $x = \frac{55}{26}e^{\frac{4}{3}t} - \frac{3}{26}\cos 2t - \frac{1}{13}\sin 2t$

(5) $x = \frac{3}{4}e^{2t} - \frac{3}{2}t - \frac{3}{5}$ (6) $\theta = (A - \alpha)e^{-kt} + \alpha$

Second order:

(1) $x = -\frac{1}{8}\cos 3t + \frac{1}{8}\cos t$ (2) $x = -\frac{1}{8}\cos 3t + \frac{9}{8}\cos t + 2\sin t$

(3) $x = \cos\frac{3t}{2} + \frac{32}{21}\sin\frac{3t}{2} - \frac{1}{7}\sin 2t$ (4) $x = \frac{1}{6}e^{t} + \frac{5}{2}e^{-t} - \frac{5}{3}e^{-2t}$

(5) $x = e^{2t} - e^{t}(t+1)$ (6) $x = e^{-t}(\sin t + \cos t)$

(7) $x = e^{-t}\left(\frac{2}{5}\cos t + \frac{1}{5}\sin t\right) - \frac{2}{5}\cos t + \frac{1}{5}\sin t$