ENGINEERING ANALYSIS

TUTORIAL 2 MATRIX ALGEBRA

1. Solve the following system of linear equations using **Gaussian elimination:**

(a) without pivotting; (b) with partial pivotting

(a)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 - 4x_2 - 2x_3 &= 2 \\ 5x_1 + 3x_2 + 5x_3 &= -1 \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 2 \\ x_1 + 3x_2 - x_3 &= 11 \\ 2x_1 - 2x_2 + 5x_3 &= 3 \end{aligned}$$

(c)
$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 3 \\ 2x_1 - x_2 - x_3 &= 11 \\ 3x_1 + 2x_2 + x_3 &= -5 \end{aligned}$$

(d)
$$\begin{aligned} x - 4y - 2z &= 21 \\ 2x + y + 2z &= 3 \\ 3x + 2y - z &= -2 \end{aligned}$$

[Answers:
(a) x_1=**2**,x_2=**3**,x_3=**-4**; (b) x_1= **-1**, x_2 = **5**, x_3=**3**; (c) x_1 =**2**, x_2= **-4**, x_3= **-3**; (d) x=**3**,y=**-5**,z=**1**]

2. Solve the following system of homogeneous linear equations by any method

(a)
$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 0 \\ 4x_1 + 5x_2 + 6x_3 &= 0 \\ 3x_1 + x_2 - 2x_3 &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 - x_3 &= 0 \\ 3x_1 - 3x_2 + 2x_3 &= 0 \\ -x_1 - 11x_2 + 6x_3 &= 0 \end{aligned}$$

(c)
$$\begin{aligned} x_1 + x_2 - x_3 &= 0 \\ 2x_1 - 4x_2 + 3x_3 &= 0 \\ -x_1 - 7x_2 + 6x_3 &= 0 \end{aligned}$$

(d)
$$\begin{aligned} 2x - y - 3z &= 0 \\ -x + 2y - 3z &= 0 \\ x + y + 4z &= 0 \end{aligned}$$

3. Using Gaussian elimination find conditions on ***a*** and ***b*** for which the system

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -1 \\ 3x_1 - x_2 + 2x_3 &= 7 \\ 5x_1 + 3x_2 + \mathbf{a}x_3 &= \mathbf{b} \end{aligned}$$

has

(a) a unique solution
(b) infinitely many solutions
(c) no solutions.