

ENGINEERING ANALYSIS

MATRIX ALGEBRA 3

1. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given form. Solve the system

$$(a) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right],$$

$$(b) \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right],$$

$$(c) \left[\begin{array}{cccc|c} 1 & 6 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right],$$

$$(d) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right].$$

2. Solve the following (where possible) by Gaussian elimination.

$$(a) \begin{cases} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \\ 3x - 3w = -3 \end{cases},$$

$$(b) \begin{cases} -2b + 3c = 1 \\ 3a + 6b - 3c = -2 \\ 6a + 6b + 3c = 5 \end{cases},$$

$$(c) \begin{cases} 4x_1 - 8x_2 = 12 \\ 3x_1 - 6x_2 = 9 \\ -2x_1 + 4x_2 = -6 \end{cases},$$

$$(d) \begin{cases} 10y - 4z + w = 1 \\ x + 4y - z + w = 2 \\ -2x - 8y + 2z - 2w = -4 \\ x - 6y + 3z = 1 \end{cases},$$

$$(e) \begin{cases} 5x_1 - 2x_2 + 6x_3 = 0 \\ -2x_1 + x_2 + 3x_3 = 1 \end{cases},$$

$$(f) \begin{cases} x_1 - 2x_2 + x_3 - 4x_4 = 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 = 2 \\ x_1 - 12x_2 - 11x_3 - 16x_4 = 5 \end{cases}.$$

3. Solve the following homogeneous systems of linear equations.

$$\begin{aligned} & 2x_1 + x_2 + 3x_3 = 0 \\ \text{(a)} \quad & x_1 + 2x_2 = 0 \\ & x_2 + x_3 = 0 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 3x_1 + x_2 + x_3 + x_4 = 0 \\ & 5x_1 - x_2 + x_3 - x_4 = 0 \end{aligned}$$

$$\begin{aligned} & 2x + 2y + 4z = 0 \\ \text{(c)} \quad & w - y - 3z = 0 \\ & 2w + 3x + y + z = 0 \\ & -2w + x + 3y - 2z = 0 \end{aligned}$$

$$\begin{aligned} & z_3 + z_4 + z_5 = 0 \\ \text{(d)} \quad & -z_1 - z_2 + 2z_3 - 3z_4 + z_5 = 0 \\ & z_1 + z_2 - 2z_3 - z_5 = 0 \\ & 2z_1 + 2z_2 - z_3 + z_5 = 0 \end{aligned}$$

Answers:

1. (a) $x_1=5, x_2=-2, x_3=4$.
 (b) $x_1=-1-4t, x_2=6-2t, x_3=2-3t, x_4=t$
 (c) $x_1=-2-4t-6s, x_2=s, x_3=1-3t, x_4=2-5t, x_5=t$
 (d) no solution.

2. (a) $x=t-1, y=2s, z=s, w=t$
 (b) *Inconsistent*
 (c) $x_1=3+2t, x_2=t$
 (d) $x = \frac{8}{5} - \frac{3}{5}t - \frac{3}{5}s, y = \frac{1}{10} + \frac{2t}{5} - \frac{1}{10}s, z = t, w = s$
 (e) $x_1=2-12t, x_2=5-27t, x_3=t$

3. (a) $x_1=0, x_2=0, x_3=0$
 (b) $x_1=-s, x_2=-t-s, x_3=4s, x_4=t$
 (c) $w=t, x=-t, y=t, z=0$
 (d) $z_1=s, z_2=-t-s, z_3=-t, z_4=0, z_5=t$