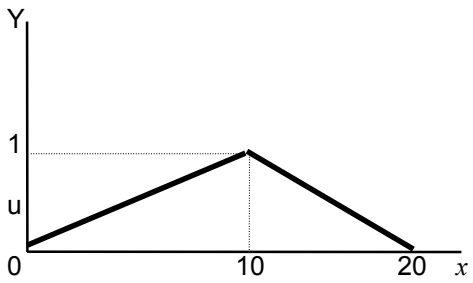
ENGINEERING MATHEMATICS

THE WAVE EQUATION TUTORIAL

1. A stretched string of length 20 cm is set oscillating by displacing its mid-point a distance 1 cm from its rest position and releasing it with zero initial velocity. Solve the wave equation



$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

where $c^2 = 1$ to determine the resulting motion, $u(x,t)$.

2. The centre point of a perfectly elastic string stretched between two points A and B, 4 m apart, is deflected a distance 0.01 m from its position of rest perpendicular to AB and released initially with zero velocity. Apply the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

where $c = 10$ to determine the subsequent motion of a point P distant x from A at time t.

3. An elastic string is stretched between two points 10 cm apart. A point P on the string 2 cm from the left-hand end, (i.e. the origin) is drawn aside 1 cm from its position of rest and released with zero velocity. Solve the one-dimensional wave equation to determine the displacement of any point at any instant.

4. If $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$ and $c = 3$, determine the solution $u = u(x,t)$ subject to the boundary conditions

$$u(0,t) = 0 \quad \text{and} \quad u(2,t) = 0 \quad \text{for} \quad t \geq 0$$

$$u(x,0) = x(2 - x) \quad \text{and} \quad \left[\frac{\partial u}{\partial t}\right]_{t=0} = 0 \qquad 0 \leq x \leq 2 .$$