Electrical and Electronic Engineering

2nd Year

Specimen Examination Paper - Semester 1 1998

Time Allowed 2 hours

Answer FOUR questions

1. (a) Evaluate the determinants

(i) $$\begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 2 & 1 & 0 \end{vmatrix}$$

(ii) $$\begin{vmatrix} 3 & -1 & 4 \\ 6 & 3 & 5 \\ 2 & -1 & 6 \end{vmatrix}$$

(b) Show that

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \\ -1 & -3 & 8 \end{vmatrix} = 0$$

(c) Find the adjoint matrices and hence the inverses (where possible) of:

(i) $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 2 & 1 & 0 \end{pmatrix}$ (ii) $B = \begin{pmatrix} 3 & -1 & 4 \\ 6 & 3 & 5 \\ 2 & -1 & 6 \end{pmatrix}$ (iii) $C = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \\ -1 & -3 & 8 \end{pmatrix}$

(d) Use the results from (a), (b), (c) to solve

(i) $$\begin{aligned} x + 3z &= 7 \\ y + 4z &= 7 \\ 2x + y &= 1 \end{aligned}$$

(ii) $$\begin{aligned} x + 2y - z &= 2 \\ 2x + 3y + 5z &= 5 \\ -x - 3y + 8z &= -1 \end{aligned}$$

by an appropriate method.

2. Solve the following equations using Gaussian Elimination <u>with partial pivoting</u>:

(i)
$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 2 \\ 3x_1 + 2x_3 - 2x_4 &= -8 \\ 4x_2 - x_3 - x_4 &= 1 \\ x_1 + 6x_2 - 2x_3 &= 7 \end{aligned}$$

(ii)
$$\begin{aligned} x + 2y - 4z &= 4 \\ -2x - 4y + 8z &= -8 \end{aligned}$$

(iii)
$$\begin{aligned} x + y - z &= 0 \\ 4x - y + 5z &= 0 \\ 6x + y + 3z &= 0 \end{aligned}$$

3. (a) Find the Laplace Transforms, in their simplest forms, of:

(i) $3t^5$ (ii) $5e^{-3t}$ (iii) $\frac{1}{2}\sin 2t$ (iv) $\frac{1}{3}\cos 9t$ (v) $e^{-2t}\sin 3t$ (vi) $e^{4t}\cos 2t$ (vii) $t^2 e^{2t}$ (viii) $t^3 e^{-4t}$

(b) Write the following as partial fractions and hence find the inverse Laplace transforms of each one

(i) $\dfrac{1}{(s+3)(s+7)}$ (ii) $\dfrac{s+5}{(s+1)(s-3)}$ (iii) $\dfrac{3s^2-7s+5}{(s-1)(s-2)(s-3)}$

(c) Using Laplace Transform methods solve, for $t \geq 0$, the following differential equations:

(i) $\dfrac{dx}{dt} + 3x = e^{-2t}$ with $x(0) = 2$

(ii) $\dfrac{d^2x}{dt^2} + \dfrac{2dx}{dt} + 5x = 1$ with $x(0) = 0, \quad \dot{x}(0) = 0$

4. (a) Show that the inverse Laplace Transform of

$$\frac{1}{(s+1)(s+2)(s^2+2s+1)} \quad \text{is} \quad e^{-t} - \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-t}(\sin t + \cos t)$$

(b) A voltage source $Ve^{-t}\sin t$ is applied across a series LCR circuit with L=1, R=3, C=$\frac{1}{2}$. Show that the current $i(t)$ in the circuit satisfies the differential equation

$$\frac{d^2 i}{dt^2} + \frac{3di}{dt} + 2i = Ve^{-t}\sin t$$

Find the current $i(t)$ in the circuit at time $t \ge 0$ if $i(t)$ satisfies the initial conditions $i(0) = 1$ and $\frac{d}{dt}i(0) = 2$.

5. Sketch the function

$$f(t) = \begin{cases} 2t & 0 < t < 2 \\ 4 & 2 < t < 4 \end{cases}$$

$$f(t+4) = f(t)$$

for $-8 \le t \le 8$.

Find the Fourier series for the function.

6. Solve the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

for the vibration of a string stretched between the points $x = 0$ and $x = 10$ and subject to the boundary and initial conditions:

(a) $u(0,t) = 0$ (fixed end at $x = 0$)

(b) $u(10,t) = 0$ (fixed end at $x = 10$)

(c) $\left.\frac{\partial u}{\partial t}\right]_{t=0} = 0$ (zero initial velocity)

(d) $u(x,0) = \begin{cases} x & 0 \le x \le 5 \\ 10 - x & 5 \le x \le 10 \end{cases}$