SHEET-1 DIFFERENTIATION

1. By taking appropriate limits find the derivative of the following

$(a) s = 1 - t^2, \left(\frac{ds}{dt}\right), \qquad (b) v = 3t^3 - 4t^2, \left(\frac{dv}{dt}\right), \qquad (c) y = \frac{1}{2}x^2 + x, \left(\frac{dy}{dx}\right).$

2. Find the gradient of the curve $y = \sqrt{x}$ at the point where $x = 9$.

3. Find the gradient of the curve $y = \frac{1}{x}$ at the point where $x = 3$.

4. Show that there is no point on the curve $y = \frac{1}{x^2}$ where the gradient is zero.

5. Find the coordinates of the two points on the curve $y = 5x^3 - 7x^2 + 3x + 2$ where the gradient is zero.

6. Show that if $x > 1$ then the gradient of the curve $y = \frac{1}{x} + x$ is always positive.

7. A beam AB has a bending moment M *(kNm)* given by the formula $M = 6x^2 - 12x$, where x is the distance from A in metres. Find the rate of change of M with respect to x. When is this rate zero?

8. The period of oscillation p of a simple pendulum is given by $p = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the pendulum (m) and g is the acceleration due to gravity (m/s^2). If the pendulum's length changes (due to a change in temperature say), its period will change. Find the rate of change of the period with respect to length when $l = 0.3$. Take $g = 9.8\,m/s^2$ and give your answer to three decimal places.

9. The kinetic energy K *(joules)* of a body of mass m *(kg)* travelling with speed v *(m/s)* is given by $K = \frac{1}{2}mv^2$. Find the rate of change of K with respect to velocity of a body of mass *10 kg* at an instant when its velocity is 25 m/.s.

10. The power P *(watts)* dissipated in a resistor of R *(ohms)* when a current of I *(amperes)* flows is given by $P = I^2R$. Find the rate of change of power with respect to current when $I = 3\,mA$ and $R = 1\,k\Omega$.

11. A body moves along a straight line AB so that its displacement s *(m)* from the point A at a time t *(s)* is given by $s = t^3 + 3$. Show that the body is initially (i.e. at $t = 0$) *3 m* from A and that it always moves away from A.

12. A particle moves so that its displacement s from a point P at a time t is given by $s = 1 + 2t - 0.1t^2$. Find the time at which its velocity is zero and show that its acceleration is always constant.