SHEET 11 APPLICATIONS OF INTEGRATION

1. Find the area enclosed between the following curves and straight lines
   (a) $y = 6x^2 + 2$ & $y = 12x + 2$, (b) $y = 2x^2 + 1$ & $y = 6x + 1$.
2. The parametric forma curve is given by $x = a(2t - \sin 2t), y = 2a\sin^2 t$.
   Show that the area enclosed by the curve from $t = 0$ and $t = \pi$ is $3\pi a^2$ units^2.
3. Find the length of the curve $x = 2\cos^3\theta, y = 2\sin^3\theta$ between the points corresponding to $\theta = 0$ and $\theta = \dfrac{\pi}{2}$.
4. Find the length of the curve $x = 5(2t - \sin 2t), y = 10\sin^2 t$ between the points corresponding to $t = 0$ and $t = \pi$.
5. The capacity of a battery is a measure of $\int i\,dt$, where i is the current. Estimate, using the Trapezium rule, the capacity of a battery whose current was measured over an eight hour period with the results shown below

Time/hours	0	1	2	3	4	5	6	7	8
Current/Amps	25.2	29.0	31.8	36.5	33.7	31.2	29.6	27.3	28.6

6. Use Simpson's rule with ten strips to estimate the following integral

$$\int_0^5 \sqrt{1 - \frac{x^2}{25}}\,dx \ .$$

   Hence estimate both the area of the ellipse $\dfrac{y^2}{4} + \dfrac{x^2}{25} = 1$ and π.
7. The speed V of a rocket at a time t after launch is given by $V = at^2 + b$, where a and b are constants. Given that the mean speed over the first 10 seconds was $10ms^{-1}$ and that over the next second was $50ms^{-1}$, find a and b. What was the mean speed over the third second?
8. When a homogeneous, isotropic elastic bar of constant cross sectional area A is under uniformly distributed tensile stress, the elongation in the direction of the tensile force P is given by $stress = E \times strain$, where E is the Young's modulus of the material, the stress is the applied force per unit area and the strain is the ratio of the elongation (e) to the unstretched length of the bar (L), i.e. $E\dfrac{e}{L} = \dfrac{P}{A}$. Now consider a bar of circular cross section, length L whose diameter varies along its length so that $A = A_0 + kx^2$, where x is the distance from the end with smallest area and therefore $kL^2 = A_1 - A_0$ where A_1 is the area of the larger end of the bar. By considering the elongation of an element of thickness Δx, or otherwise, show that the total elongation of the bar under tensile force P is

$$l = \int_0^L \frac{P\,dx}{E(A_0 + kx^2)}.$$

   Show further that $l = \dfrac{4PL}{\pi d_0 d_2}\cos^{-1}\left(\dfrac{d_0}{d_1}\right)$, where d_0 and d_1 are the end diameters of the bar, $d_0 < d_1$ and $d_2^{\,2} = d_1^{\,2} - d_0^{\,2}$.