SHEET-3 PRODUCT RULE AND QUOTIENT RULE

1. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in each of the following

$(a)\ y = x^3 \sin x, \qquad (b)\ y = 4x^5 \cos x, \qquad (c)\ y = 2x^2 \sin^2 x,$

$(d)\ y = 3e^{3x}(\sin^2 x + 3\cos^2 x), \qquad (e)\ y = (x+1)^3(2x-1)^4.$

2. Find $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ in each of the following

$(a)\ x = (t+1)^2 \cos^4 t, \ (b)\ x = t\ln(t^2+1), \qquad (c)\ x = 4t^3 \sin t \cos t,$

$(d)\ x = 3e^{3t}(\sin^2 t + 3\cos^2 t), \qquad (e)\ x = (t^2+1)^4(\sin t + 1)^3.$

3. Find $\frac{ds}{dt}$ in each of the following

$(a)\ s = \frac{e^t - e^{-t}}{e^t + e^{-t}}, \qquad (b)\ s = \frac{\sin(\omega t + \alpha)}{e^t}, \quad (c)\ s = \frac{e^{at}}{t}.$

4. A particle moves so that its displacement along the x axis is given by $s = 3e^{-3t}\sin 2t$. Find its velocity and acceleration at time t, and show that

$$\frac{d^2s}{dt^2} + 6\frac{ds}{dt} + 13s = 0.$$

Finally, show that the following displacements also satisfy the above differential equation

(a) $s = e^{-3t} cos\, 2t,$

(b) $x = e^{-3t} sin(2t + \alpha),$ where α is a constant.

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in each of the following

$(a)\ y = \frac{x^2 + 3x - 1}{x^3 + 1}, \qquad (b)\ y = \frac{\sin^2 x + 2\cos^2 x}{3x + 1}.$