SHEET-4 MAXIMUMS AND MINIMUMS

1. Find the stationary values of the following function and classify them as either maximum or minimum.

$$y = \frac{6x^3}{3} + \frac{5x^2}{2} - 4x + 1.$$

2. Find the stationary values of the following function and classify them as either maximum or minimum.

$$y = 2x^3 - 5x^2 + 4x - 1.$$

3. The power transmitted to a belt drive is proportional to $Tv - \frac{wv^3}{g}$, where v is the speed of the belt, T the tension on the driving side, w the weight per unit length and g the acceleration due to gravity. Find the speed at which the transmitted power is a maximum.

4. The power dissipated in an electrical circuit is given by $P = I^2R$ Watts, where I is the current flowing (amperes) and R is the resistance (ohms). A circuit consists of two resistors R_1 and R_2 in parallel. The sum of the two resistors is $100K\Omega$ and a constant current of $2mA$ flows in the circuit. What is the maximum power? You may use the fact that the total resistance R_T is given by

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}.$$

5. The fuel economy E of a car in miles per gallon is given by

$$E = 35 + 2.07 \times 10^{-2} v^2 - 3.85 \times 10^{-6} v^4,$$

where v is the speed in miles per hour $(5 \le v \le 70)$. What is the most economical fuel consumption and at what speed is it achieved?

6. A contractor has $150m$ of security fencing and wishes to enclose a rectangular area. This area is divided into two rectangular parts with fencing. The two areas A_1 and A_2 are such that $A_1 = 2A_2$. What are the dimensions of the enclosure so that the total area is a maximum.

7. Two oil pipes with circular cross-section are such that the sum of their diameters must not exceed $1m$. What should their diameters be so to give the maximum possible combined cross-sectional area?

8. The signalling range of a submarine cable is proportional to $r^2 \ln\left(\frac{1}{r}\right)$, where r is the ratio of the radii of the conductor and cable, Find the value of r for the maximum range.

9. The motion of a particle performing damped vibrations is given by $x = e^{-t} \sin 2t$, x being the displacement from its mean position at time t. Show that x is a maximum at time $t = \frac{1}{2}\tan^{-1}(2)$ and determine this maximum displacement to 3 significant figures.

10. The velocity (v) of a piston is related to the angular frequency (ω) of the crank by the relationship

$$v = \omega r\left\{\sin\theta + \frac{r}{2l}\sin 2\theta\right\},$$

where r is the length of the crank and l is the length of the connecting rod. Find the first positive value of θ for which v is a maximum in the case when $l = 4r$.

11. A right circular cone of base radius r has total surface area S and volume V. Prove that

$$9V^2 = r^2\left(S^2 - 2\pi r^2 S\right).$$

If S is constant prove that the vertical angle θ of the cone of maximum volume is given by

$$\theta = 2\sin^{-1}\left(\frac{1}{3}\right).$$

12. Show that the equation

$$4\frac{d^2x}{dt^2} + 4\mu\frac{dx}{dt} + \mu^2 x = 0$$

is satisfied by $x = (At + B)e^{-\frac{\mu t}{2}}$, where A and B are arbitrary constants.
If $x = 0$ and $\frac{dx}{dt} = C$, when $t = 0$, find A and B and show that the maximum value of x is $\frac{2C}{\mu e}$ and occurs when $t = \frac{2}{\mu}$.