SHEET-5 FURTHER DIFFERENTIATION

1. In the following equations find $\frac{dx}{dt}$

$(a) x = 3\sin(2t), \quad (b) x = \tan\left(\frac{\pi}{4} - \frac{t}{2}\right), \quad (c) x = \sin^2 t + \cos^2 t,$

$(d)\ x = t^2 e^{-2t}\sin(2t), \quad (e)\ x = \frac{\cos(2t)}{t^2}, \quad (f)\ x = \sin(2t)\cos(3t).$

2. Differentiate the following with respect to x

$(a)\ln\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right), \quad (b)\ln(\sec x + \tan x), \quad (c)\sin^4 x \cos^3 x.$

3. If $t^2 + x^2 - 2t + 2x = 23,$ find $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ at the point where $t = -2, x = 3.$

4. Differentiate the following

$(a) y = x^2\cos^2 x, \quad (b) x = \ln\left(t^2\sqrt{1-t^2}\right), \quad (c) y = \frac{e^{2x}\ln x}{(x-1)^3}.$

5. If $(x-y)^3 = A(x+y),$ prove that $(2x+y)\frac{dy}{dx} = x + 2y.$

6. If $x = 3(1-\cos\theta)$ and $y = 3(\theta - \sin\theta)$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in their simplest forms.

7. If $x = \ln\left(\tan\frac{\theta}{2}\right)$ and $y = \tan\theta - \theta,$ prove that

$$\frac{d^2y}{dx^2} = \tan^2\theta\sin\theta(\cos\theta + 2\sec\theta).$$

8. If $x = e^{2t}\cos(2t-3),$ verify that $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 8x = 0.$

9. The parametric equations of a curve are given by $x = \cos 2\theta, y = 1 + \sin 2\theta.$ Find both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. Find also the equation of the curve as a relationship between x and y and find both the first and second derivatives by differentiating implicitly.

10. If $y = \left(x + \sqrt{1+x^2}\right)^{\frac{3}{2}}$, show that $4(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 9y = 0.$

11. If $x = 3\cos\theta - \cos^3\theta, y = 3\sin\theta - \sin^3\theta,$ express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $\theta.$

12. Show that $y = e^{-2mx}\sin 4mx$ is a solution of the equation

$$\frac{d^2y}{dx^2} + 4m\frac{dy}{dx} + 20m^2y = 0.$$

13. Prove that $x = Ae^{-kt}\sin pt,$ satisfies the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + (p^2 + k^2)x = 0.$$

14. Find all the maximums and minimums values of the following functions and sketch their graphs

(a) $y = x^4 - 2x^3 - 2x^2,$ (b) $y = x^2e^{-x},$ (c) $y = \frac{x}{1-x^2},$

(d) $y = \frac{x+3}{(x-1)(x+2)},$ (e) $y = \frac{(2x+1)(x-1)}{x+1}.$

15. Use Logarithmic differentiation to find the derivatives of the following functions:

(a) $y = \frac{(x-1)^2(2x-1)^3}{x+2},$ (b) $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}},$ (c) $y = x^{\sin x},$

(d) $y = (\cos x)^x.$