SHEET-6 PARTIAL DIFFERENTIATION

1. Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ in each of the following cases

(a) $f(x,y) = 1 + x + y$, (b) $f(x,y) = x^2 + 2y^2 + 3xy - x + 1$,

(c) $f(x,y) = \sin(x - y)$, (d) $f(x,y) = \frac{y}{x}$,

(e) $f(x,y) = e^{2x+3y}$, (f) $f(x,y) = \frac{1}{x} + \frac{1}{y}$.

2. If $z = e^x(x\cos y - y\sin y)$ show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

3. The displacement y of a point on a vibrating stretched string at a point x from one end at a time t is known to satisfy the partial differential equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

Show that one possible solution of this equation is given by

$$y = \sin\left(\frac{px}{c}\right)\sin(pt + a),$$

where p, c and a are constants.

4. Show that $\Phi = e^{-\frac{kt}{2}} \sin pt \cos qx$ satisfies the equation

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{1}{c^2}\left\{\frac{\partial^2 \Phi}{\partial t^2} + k\frac{\partial \Phi}{\partial t}\right\}$$

provided $p^2 = c^2 q^2 - \frac{k^2}{4}$.

5. The coefficient of rigidity (n) of a wire of length (L) and uniform diameter (d) is given by $n = \frac{AL}{d^4}$, where A is a constant. If errors of $\pm 0.25\%$ and $\pm 1\%$ are possible in measuring L and d, respectively, determine the maximum percentage error in calculating the value of n.

6. The deflection y at the centre of a circular plate suspended at the edge and uniformly loaded is given by $y = \frac{kwd^4}{t^3}$, where w is the total load, t the thickness, d the diameter and k a constant. Calculate the approximate percentage change in y if w is increased by 3%, d is decreased by 2.5% and t is increased by 4%.