SHEET-7 FUTHER PARTIAL DIFFERENTIATION

1. Show that the following functions satisfy Laplace's equation in two dimensions, viz. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$

   $(a) \quad V = e^{-kx}(A\cos ky + B\sin ky), \quad (b) V = \ln(x^2 + y^2), \quad (c) V = \tan^{-1}\left(\frac{x}{y}\right).$

2. Each of the following functions V is a function of x and y, which are themselves functions of t. Determine $\frac{dV}{dt}$ by the chain rule. In each case check your answer by substituting for x and y in terms of t and finding $\frac{dV}{dt}$ directly.

   $(a)\ V(x, y) = 3x + xy - y^2,\ x = \cos 3t, y = \sin 3t,$

   $(b)\ V(x, y) = (x^2 - 1)\sqrt{y}, x = \cosh t, y = \sinh^2 t.$

3. If $v = \frac{xy}{(x^2 + y^2)^2}$ and $x = r\cos\theta, y = r\sin\theta,$ show that

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0.$$

4. The base radius *r cm* of a right circular cone increases at *2 cm/s* and its height *h* at *3 cm/s*. Find the rate of increase of the volume of the cone when $r = 5, h = 15$.

   Note: The volume of the cone is $\frac{1}{3}\pi r^2 h.$

5. The two equal sides and the inlcuded angle of an isosceles triangle are increasing at rates of *0.5 metres/hour* and $2^o / hour$, respectively. How fast is the area of the triangle increasing when the length of the two equal sides is *10 metres* and the included angle is 30^o.

6. If $Z = x^2 y^3$, find the maximum percentage error in z if x and y are subject to errors of $\pm 3\%$ and $\pm 2\%$, respectively.

7. The lengths of the two shorter sides of a right-angled triangle are measured as *17cm* and *23cm.* If each measurement has an error of $\pm 0.1cm$, find the maximum error in the calculated value of the hypotenuse.